Static configurations of gravitating dusty plasmas

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(Received 10 July 2002; published 11 November 2002)

Plasma and dust are key ingredients of the universe, and in larger astrophysical systems the gravitational and electrostatic forces can become comparable, thus prompting a careful revision of stationary configurations of self-gravitating dusty plasmas. An overview of some physically acceptable models is given, with applications to relevant structures in space and astrophysical plasmas. For those cases where explicit solutions can be obtained, the scales over which the system is inhomogeneous are of the order of the Jeans lengths, determined in the usual way by studying local perturbations of a uniform equilibrium.

DOI: 10.1103/PhysRevE.66.056404 PACS number(s): 52.27.Lw, 95.30.Qd, 98.38.Cp

I. INTRODUCTION

In this paper results are combined from two physical domains, dusty plasma physics and self-gravitation of extended mass distributions. One of the main constituents of the universe is dust, a loose term that covers particles that are large compared to atoms and ionized nuclei and with compositions that can greatly vary with the astrophysical context. The presence of (neutral) dust has been known for a long time from remote astronomical observations, as for the dust around and between stars [1–3]. Successful space missions to planets and comets have brought compelling evidence for the necessity of charged dust, however, as grains immersed in ambient plasmas and radiative environments become electrically charged by a variety of processes, the most simple ones being capture of plasma electrons and ions or photoionization. Further discussions about the many fascinating properties of dusty plasmas can be found in recent reviews [4,5] and books [6–8], and in the references contained therein.

Extended systems of mass particles have originally been studied as neutral, uncharged matter, for which electromagnetic effects need not be considered. Contrary to electromagnetic forces, gravitation is always attractive and cannot be shielded, so that gravitational collapse of large regions with distributed masses is inevitable, unless counteracted by thermal agitation or repulsion due to other causes. In most cases a truly homogeneous equilibrium is not possible, and the determination of self-consistent but nonhomogeneous stationary configurations remains in many cases a vexing problem [9–12]. The inherent difficulty of finding self-consistent solutions is sometimes circumvented by studying only local perturbations, whose wavelengths are small compared to the inhomogeneity scale lengths. The resulting instability has been studied in the astrophysical literature for a century, since Jeans obtained in 1902 the criterion that now bears his name, predicting the size of extended mass systems that will gravitationally collapse. This approach starts from an unperturbed gaseous cloud that is initially supposed to be completely uniform [13]. In itself, there is nothing wrong with this procedure, were it not that the locality condition can seldom be tested *a posteriori*, because knowledge about the true equilibrium is lacking!

In certain dusty plasmas the gravitational effects can become large enough to matter in the full description, hence the need for a careful discussion of possible stationary states in dusty plasmas with self-gravitation. This results in a significant modification of collective modes and in new stability conditions. Many approximations occur in dealing with the low frequencies at which the dust manifests itself, with the small gravitational effects between charged dust particles and with the correct balance between electromagnetic and gravitational forces [7]. It is thus necessary to treat dusty self-gravitational plasmas with great care, as we specifically propose to do here for some stationary states with Cartesian one-dimensional symmetry, without mass flows.

Our paper is structured as follows. We first discuss in Sec. II static and stationary states of a single charged fluid in the ideal magnetohydrodynamic (MHD) approximation, before turning in Sec. III to a multispecies plasma, together with indications about the scale lengths on which the system varies. More specific comments about how to translate this information in the case of a usual dusty plasma is given in Sec. IV. It turns out that for those cases where explicit solutions can be obtained, the inhomogeneity length scales are of the order of the Jeans lengths, and we give in Sec. V a feeling for the orders of magnitude for interstellar dust clouds. Finally, our conclusions are recalled in Sec. VI.

II. MAGNETOHYDRODYNAMIC DESCRIPTION

Because there are already so many possibilities to discuss stationary states of neutral gases, we will start with the simplest extension to charged fluids, whereby all particles are considered together in a nonrelativistic MHD description without dissipation. It is worth recalling that ideal MHD is the simplest, internally self-consistent description of charged fluids [14]. All charge separation disappears, and elimination of the electric field leaves us with the reduced set of ideal MHD equations of continuity, induction and momentum, augmented here by the gravitational Poisson's equation,

$$\frac{\partial}{\partial t}\rho+\nabla\cdot(\rho\mathbf{u})=0,$$

$$\frac{\partial}{\partial t}\mathbf{B}=\nabla\times(\mathbf{u}\times\mathbf{B}),$$

$$\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u}+\frac{1}{\rho}\nabla p+\nabla\psi=\frac{1}{\mu_0\rho}(\nabla\times\mathbf{B})\times\mathbf{B},$$

$$\nabla^2\psi=4\pi G\rho. \tag{1}$$

Here ρ, $\mathbf{u}$, and p refer to the charged fluid mass density, velocity, and pressure, respectively, $\mathbf{B}$ to the magnetic induction, ψ to the self-gravitational potential, G is the gravitational and μ_0 the vacuum magnetic permeability constant.

In this section we review static, Cartesian one-dimensional models. There are many others that might be considered and are mathematically admissible, but not of equal physical interest. Flows will be left out of the description, because it is difficult to envisage a realistic Cartesian one-dimensional astrophysical situation to which this might apply. Indeed, if mass would flow in one direction, there would be depletion or a source at one end and an accumulation or sink at the other, and this cannot be stationary. For the same reason flows with spherical symmetry would lead to a total pile-up or depletion at the center of symmetry. Moreover, in spherically symmetric states magnetic fields cannot be included, since these would break the spherical symmetry, in view of Gauss law. How these results can be extended to the dusty plasmas that we have in mind as our ultimate goal will be discussed in Sec. IV.

Extending our preliminary results [15] in the absence of flows, the equation of motion from Eq. (1) reduces in the stationary state $(\partial\cdot/\partial t=0)$ to

$$\nabla\left(p_0+\frac{B_0^2}{2\mu_0}\right)+\rho_0\nabla\psi_0=\frac{1}{\mu_0}(\mathbf{B}_0\cdot\nabla)\mathbf{B}_0. \tag{2}$$

Subscripts 0 refer to the stationary state. If $\mathbf{B}_0$ has a component in the direction of the gradients in Eq. (2), then the whole equilibrium field is strictly constant everywhere and consequently disappears from the description. Otherwise, the right-hand side of Eq. (2) vanishes but the gradient of B_0^2 remains on the left-hand side. Since the latter is a more general picture, we will continue with it.

To relate the information obtained so far to Poisson's gravitational law, we need to introduce some hypotheses about the pressure and about the magnetic field. We take a polytropic pressure, where $p_0=C\rho_0^\gamma$, with γ the polytropic index, and assume the magnetic field to vary in such a way that the ratio β of the plasma pressure to the magnetic pressure $B_0^2/2\mu_0$ remains constant. In this model the Alfvén speed V_A is constant, and both the magnetic field and plasma pressures are stronger in denser regions. This is rather realistic and may occur in highly conductive plasmas with frozen-in magnetic fields [15]. The magnetohydrostatic equilibrium balance equation thus reduces to

$$\frac{1+\beta}{\beta}\nabla p_0+\rho_0\nabla\psi_0=0. \tag{3}$$

Together with Poisson's equation in Eq. (1), this yields a single equation for ρ_0,

$$\rho_0\nabla^2\rho_0+(\gamma-2)(\nabla\rho_0)^2+\frac{4\pi G\beta}{C\gamma(1+\beta)}\rho_0^{4-\gamma}=0. \tag{4}$$

For Cartesian one-dimensional systems this becomes

$$\rho_0\frac{d^2\rho_0}{dz^2}+(\gamma-2)\left(\frac{d\rho_0}{dz}\right)^2+\frac{4\pi G\beta}{C\gamma(1+\beta)}\rho_0^{4-\gamma}=0. \tag{5}$$

For the isothermal case, with $\gamma=1$ and $C=c_s^2$ representing the sound speed squared, an analytical solution to Eq. (5) is easily obtained. For physical reasons we search for densities having an extremum at $z=0$, where the gravitational force vanishes. This leads to

$$\rho_0(z)=\rho_{00}\,\mathrm{sech}^2(z/\lambda_{ms}), \tag{6}$$

where the typical scale length λ_{ms} is defined by $\lambda_{ms}^2=(V_A^2+2c_s^2)/4\pi G\rho_{00}=(V_A^2+2c_s^2)/\omega_{Jd}^2$, ρ_{00} is the density at the center of the cloud, and the (central) Jeans frequency ω_{Jd} is given through $\omega_{Jd}^2=4\pi G\rho_{00}$.

We note that the inhomogeneity scale length is closely related to the Jeans length for magnetosonic modes, obtained from linear stability analysis in a homogeneous medium [16], hence the subscript ms to characterize this typical length. Nevertheless, it is worth repeating that we have not used the Jeans swindle, because the present treatment is fully compatible with all equilibrium constraints. Whether it is a coincidence or not that the inhomogeneity and the Jeans lengths are very similar in structure and of the same order remains food for thought or further investigation.

Turning now to the magnetic field strength, we obtain

$$B_0(z)=B_{00}\,\mathrm{sech}(z/\lambda_{ms}), \tag{7}$$

as V_A is a constant and the central field strength B_{00} is given through $B_{00}^2=\mu_0V_A^2\rho_{00}$.

For general values of γ we find the implicit solution of Eq. (5) as

$$\int_{\rho_0/\rho_{00}}^1\frac{r^{\gamma-2}\,dr}{\sqrt{1-r^\gamma}}=\frac{2}{\gamma}\frac{z}{\lambda_{ms}}. \tag{8}$$

This has been numerically investigated, for different values of γ, as illustrated in Fig. 1.

For all values of $\gamma>1$ such models turn out to be well behaved but of finite extent $z_{\max}$, determined from

$$\int_0^1\frac{r^{\gamma-2}\,dr}{\sqrt{1-r^\gamma}}=\frac{2}{\gamma}\frac{z_{\max}}{\lambda_{ms}}. \tag{9}$$

This might be an indication that fragmentation could occur, but this interesting suggestion warrants further investigation outside the scope of the present paper. Moreover, the density

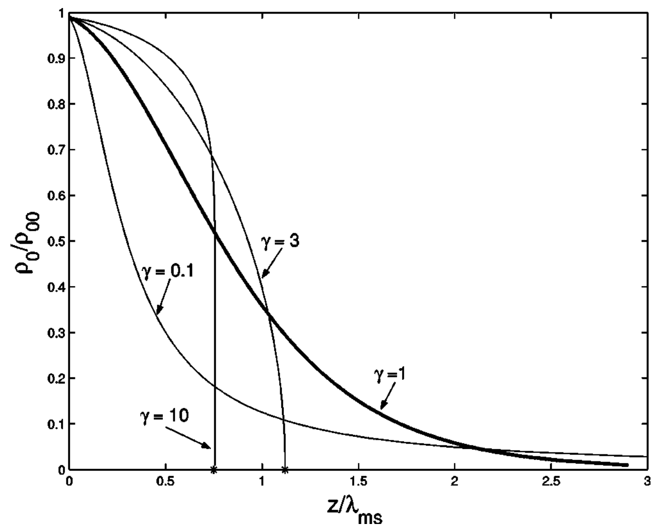

FIG. 1. Density distributions vs distance from the central location for various values of γ.

fall-off near the maximum central value is then so abrupt that local acceptable perturbations would have to be on small scales that are not compatible with the usual Jeans approach. To the contrary, models with $\gamma<1$ have an unphysical temperature behavior that increases outwardly from the center without bounds.

At the end of this section, we point out that the isothermal model is the only infinite one to possess a finite mass per unit cross-section, $2\lambda_{ms}\rho_{00}$. The models with $\gamma>1$ have a finite extent and are per force of finite mass per unit cross section.

III. MULTIFLUID CONFIGURATIONS

Our eventual aim is a study of self-gravitation in dusty plasmas, for which a multispecies description is needed. Given the long scales involved in gravitational problems that are basically macroscopic in nature, we can use a multifluid approach. The basic equations per species with label α include the continuity and momentum equations [7]

$$\frac{\partial}{\partial t} n_\alpha + \nabla\cdot(n_\alpha \mathbf{u}_\alpha)=0,$$

$$\left(\frac{\partial}{\partial t}+\mathbf{u}_\alpha\cdot\nabla\right)\mathbf{u}_\alpha+\frac{1}{n_\alpha m_\alpha}\nabla p_\alpha=\frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{u}_\alpha\times\mathbf{B})-\nabla\psi. \tag{10}$$

Here n_α, $\mathbf{u}_\alpha$, p_α, q_α, and m_α now refer to the number density, fluid velocity, pressure, charge, and mass of the respective constituents. In addition, there is the gravitational Poisson's equation

$$\nabla^2\psi=4\pi G\sum_\alpha n_\alpha m_\alpha, \tag{11}$$

and the electric field $\mathbf{E}$ and $\mathbf{B}$ obey Maxwell's equations

$$\nabla\times\mathbf{E}+\frac{\partial}{\partial t}\mathbf{B}=\mathbf{0},$$

$$c^2\nabla\times\mathbf{B}=\frac{\partial}{\partial t}\mathbf{E}+\frac{1}{\varepsilon_0}\sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha,$$

$$\nabla\cdot\mathbf{E}=\frac{1}{\varepsilon_0}\sum_\alpha n_\alpha q_\alpha, \tag{12}$$

where ε_0 is the vacuum dielectric constant.

To close the set of basic equations, we need to assume something about the pressure behavior. For simplicity we restrict ourselves to barotropic pressures, where the pressure of each species depends only on its density $p_\alpha(n_\alpha)$, which includes polytropic and isothermal pressures. Neutral gases can be included by simply turning off the charges for the relevant constituents.

The basic equations (10) indicate that for stationary states without flows the magnetic field decouples from the other physical quantities, and has a vector potential that is a harmonic function. Hence, although a stationary magnetic field is possible it need not be dealt with at this stage, since it does not influence the basic configuration. Furthermore, the equilibrium electric field is electrostatic and derivable from an electrostatic potential φ. The basic set of equations reduces to

$$\frac{1}{n_{\alpha 0}}\nabla p_{\alpha 0}+q_\alpha\nabla\varphi_0+m_\alpha\nabla\psi_0=0,$$

$$\nabla^2\varphi_0+\frac{1}{\varepsilon_0}\sum_\alpha n_{\alpha 0}q_\alpha=0,$$

$$\nabla^2\psi_0=4\pi G\sum_\alpha n_{\alpha 0}m_\alpha, \tag{13}$$

and it is through the relation between pressures and densities that the set is closed. Taking as in Sec. II a polytropic pressure $p_{\alpha 0}=C_\alpha n_{\alpha 0}^{\gamma_\alpha}$, where γ_α is the polytropic index for species α, gives for generic values of $\gamma_\alpha\neq 1$ after one integration a Bernoulli type result

$$\frac{C_\alpha\gamma_\alpha}{\gamma_\alpha-1}(n_{\alpha 0}^{\gamma_\alpha-1}-n_{\alpha 00}^{\gamma_\alpha-1})+q_\alpha\varphi_0+m_\alpha\psi_0=0. \tag{14}$$

Densities $n_{\alpha 00}$ have been specified as constant values at some reference location, preferably at the center of the cloud where, e.g., the gravitational force vanishes and both potentials φ_0 and ψ_0 reach an extremum, rescaled as 0. For isothermal constituents with $\gamma_\alpha=1$, Eq. (14) is replaced by

$$\kappa T_\alpha \ln\frac{n_{\alpha 0}}{n_{\alpha 00}}+q_\alpha\varphi_0+m_\alpha\psi_0=0, \tag{15}$$

where κ is Boltzmann's constant and T_α the species temperature. Pursuing the isothermal case further, the density can easily be expressed in terms of the potentials,

$$n_{\alpha 0}=n_{\alpha 00}\exp\left(-\frac{q_\alpha\varphi_0+m_\alpha\psi_0}{\kappa T_\alpha}\right). \tag{16}$$

Two coupled Poisson's equations remain to be solved for the potentials

$$\nabla^2\varphi_0 = -\sum_\alpha \frac{\omega_{p\alpha}^2 m_\alpha}{q_\alpha}\exp\left(-\frac{q_\alpha\varphi_0+m_\alpha\psi_0}{\kappa T_\alpha}\right),$$

$$\nabla^2\psi_0 = \sum_\alpha \omega_{J\alpha}{}^2 \exp\left(-\frac{q_\alpha\varphi_0+m_\alpha\psi_0}{\kappa T_\alpha}\right). \tag{17}$$

The respective plasma ($\omega_{p\alpha}$) and Jeans ($\omega_{J\alpha}$) frequencies have been defined by using $n_{\alpha 00}$, so that $\omega_{p\alpha}^2 = n_{\alpha 00}q_\alpha^2/\varepsilon_0 m_\alpha$ and $\omega_{J\alpha}^2 = 4\pi G n_{\alpha 00} m_\alpha$, respectively. One has to be very careful by not automatically defining, e.g., $\omega_{p\alpha}$ in terms of $n_{\alpha 0}$ and forgetting that this becomes location dependent for nonhomogeneous configurations. It is clear that analytic solutions of the coupled set (17) can only be discussed when simplifying assumptions help to reduce the mathematical complexity. This will be attempted for the usual dusty plasma model in Sec. IV.

Another way of looking at the problem is by not immediately integrating the individual equations of motions but adding them first, so that the Poisson's equations can be used in

$$\sum_\alpha \nabla p_{\alpha 0} + \left(\sum_\alpha n_{\alpha 0}q_\alpha\right)\nabla\varphi_0 + \left(\sum_\alpha n_{\alpha 0}m_\alpha\right)\nabla\psi_0 = 0 \tag{18}$$

to replace the total charge and mass densities. For strictly one-dimensional states this can be integrated to obtain a global Bernoulli type result,

$$\frac{1}{2\varepsilon_0}\left(\frac{d\varphi_0}{dz}\right)^2 - 2\pi G\left(\frac{d\psi_0}{dz}\right)^2 + \sum_\alpha p_{\alpha 0}(\varphi_0,\psi_0) = \mathcal{E}. \tag{19}$$

Since all $p_{\alpha 0}$ depend on φ_0 and ψ_0 through n_α, Eq. (19) can be viewed as the energy integral for a system with an equivalent kinetic energy that is not positive definite! Unfortunately, the mechanics literature is almost silent on what happens in such situations, which again shows that the inclusion of gravitation is a much more drastic change than the simple discussion of Jeans-like instabilities would lead one to believe.

IV. DUSTY PLASMAS

In order to see the implications for dusty plasmas and at the same time keep some of the complications that always crop up in this class of problems under control, we use the standard model for low-frequency dusty plasma phenomena, and follow the lines of recent treatments [17,18].

The simplifications on the plasma side are that the electrons and ions are massless: their inertia can be neglected because the physical phenomena occur on such a slow time scale that instant balance is achievable between the various forces. The electron and ion densities derived in Eq. (16) now reduce to standard Boltzmann expressions as if gravitation were absent,

$$n_{e0} = n_{e00}\exp\left(\frac{e\varphi_0}{\kappa T_e}\right),$$

$$n_{i0} = n_{i00}\exp\left(-\frac{e\varphi_0}{\kappa T_i}\right). \tag{20}$$

On the other hand, the charged dust is treated as monodisperse, all grains having average charge $q_d = -Z_d e$ and mass m_d, where Z_d is the charge number in absolute value.

It is tempting to assume charge neutrality to be almost perfect at all locations [17,18], so that the two coupled Poisson's equations (17) can be treated in a different way. Following briefly this line of thought, the electrostatic Poisson's equation would reduce to

$$n_{i00}\exp\left(-\frac{e\varphi_0}{\kappa T_i}\right) - n_{e00}\exp\left(\frac{e\varphi_0}{\kappa T_e}\right) - n_{d00}Z_d\exp\left(\frac{Z_d e\varphi_0 - m_d\psi_0}{\kappa T_d}\right) \simeq 0. \tag{21}$$

The physical interpretation is that the electrons and ions are tied to the charged dust by electrostatic forces. As a consequence, φ_0 becomes expressible as an algebraic function of ψ_0, or vice versa. What remains of the other Poisson's equation will reduce to an equation in ψ_0 alone when the information from Eq. (21) is inserted. Since the dust is the only constituent of the plasma mixture to feel any gravitation, that is how the mass gets distributed. The electrons and ions follow suit through their electrostatic coupling to the charged dust.

To see this more clearly, it is instructive to go to a naive model, that can be easily discussed throughout. We assume that all free electrons have been accreted on the dust grains ($n_{e00}=0$), that the ions are massless and the dust is so heavy that its temperature effects can be neglected, rendering it effectively cold. In other words, momentum balance for the ions leads to the Boltzmann expression for the ion density given in Eq. (20), but it is now dust momentum balance that relates the gravitational and the electrostatic potentials,

$$\psi_0 = \frac{Z_d e}{m_d}\varphi_0 = \frac{c_{da}^2 n_{i00}}{Z_d n_{d00}}\tilde{\varphi}_0. \tag{22}$$

The normalization on the electrostatic potential is $\tilde{\varphi}_0 = e\varphi_0/\kappa T_i$. The Debye length λ_D, defined here through the ion quantities alone, $\lambda_D^2 = \varepsilon_0\kappa T_i/n_{i00}e^2$, has been used to introduce the dust-acoustic velocity $c_{da} = \lambda_D\omega_{pd}$. The latter typifies the dust-acoustic mode [19], the paradigm of low-frequency electrostatic modes in dusty plasmas. The two Poisson's equations (17) thus become

$$\lambda_D^2\nabla^2\tilde{\varphi}_0 = \frac{Z_d n_{d0}}{n_{i00}} - \exp[-\tilde{\varphi}_0],$$

$$\lambda_{da}^2\nabla^2\tilde{\varphi}_0 = \frac{Z_d n_{d0}}{n_{i00}}. \tag{23}$$

Here a modified Jeans length has been defined, $\lambda_{da}=c_{da}/\omega_{Jd}$, expressed not with the help of the dust thermal but through the dust-acoustic velocity.

Elimination of the dust density n_{d0} between the two Poisson's equations (23) gives a single equation for the electrostatic potential $\tilde{\varphi}_0$,

$$(\lambda_{da}^2-\lambda_D^2)\nabla^2\tilde{\varphi}_0=\exp[-\tilde{\varphi}_0]. \quad (24)$$

On the other hand, elimination of the Laplacians determines a simple relation between the dust and the ion densities,

$$n_{d0}=\frac{\lambda_{da}^2}{\lambda_{da}^2-\lambda_D^2}\frac{n_{i0}}{Z_d}. \quad (25)$$

Consequently, the deviation from strict charge neutrality turns out to be

$$n_{i0}-Z_d n_{d0}=-\frac{\lambda_D^2}{\lambda_{da}^2-\lambda_D^2}n_{i0}, \quad (26)$$

indicating that at the central location there will be more negative dust than positive ions, owing to the way the self-gravitation tends to concentrate the dust, which is opposed by the electrostatic coupling to the ions. Note that our results contradict the hypothesis that the plasma could be charged neutral overall. This once more shows the pitfalls of simple minded approaches to self-gravitation. Nevertheless, the total charge density remains very small if $\lambda_D \ll \lambda_{da}$.

The solution of Eq. (24) in the one-dimensional case leads to the ion density being given by

$$n_{i0}=n_{i00}\,\mathrm{sech}^2\frac{\tilde{z}}{\sqrt{2}}. \quad (27)$$

Here $\tilde{z}$ is a dimensionless coordinate, defined by $z^2=\tilde{z}^2(\lambda_{da}^2-\lambda_D^2)$, and it has been assumed that $\lambda_{da}>\lambda_D$, which is the case for almost all dusty plasmas under study. Indeed, it is straightforward to show that $\lambda_{da}^2/\lambda_D^2=\omega_{pd}^2/\omega_{Jd}^2$ is independent of the dust density but only involves the normalized dust charge-to-mass ratio. Except for scaling factors, density profiles like Eq. (27) follow the same behavior as what was obtained in the single fluid approximation (6), but with a different definition of the inhomogeneity lengths. The charge separation is indeed strongest at the center and vanishes at infinity, as shown in Fig. 2.

The opposite cases ($\lambda_{da}<\lambda_D$ or $\omega_{pd}<\omega_{Jd}$) yield discontinuities in density, showing that very heavy charged dust cannot be stabilized by electrostatic forces alone. This is not really surprising, because the self-gravitation of the charged dust is now so strong that a physically acceptable configuration is not possible: the warm massless ions cannot counterbalance this. Again we have here a possibility of fragmentation that merits to be treated in more detail.

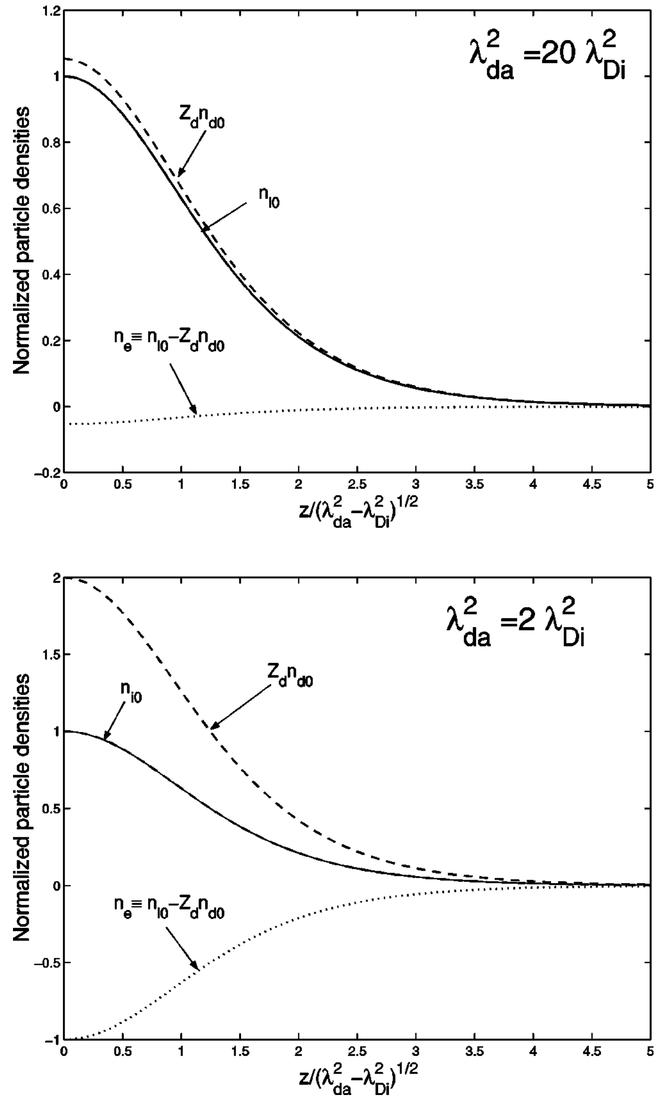

FIG. 2. Density and charge distributions vs distance from the central location, based on the solutions (27).

Lest one thinks that it is the neglect of free electrons that causes the problems with deviations from local charge neutrality, go back to the two Poisson's equations (17), use Eq. (22) for cold dust to relate ψ_0 and φ_0, and get

$$\varepsilon_0\nabla^2\varphi_0=e(n_{e0}-n_{i0})+Z_d e n_{d0},$$
$$\nabla^2\varphi_0=4\pi G n_{d0}m_d. \quad (28)$$

Elimination of the Laplacians expresses the dust density in function of the electron and the ion densities,

$$n_{d0}=\frac{\omega_{pd}^2}{\omega_{pd}^2-\omega_{Jd}^2}\frac{n_{i0}-n_{e0}}{Z_d}=\frac{\lambda_{da}^2}{\lambda_{da}^2-\lambda_D^2}\frac{n_{i0}-n_{e0}}{Z_d}, \quad (29)$$

so that there is always a deviation from local charge neutrality, since

$$n_{e0}-n_{i0}+Z_d n_{d0}=-\frac{\lambda_D^2}{\lambda_{da}^2-\lambda_D^2}\,(n_{i0}-n_{e0}). \tag{30}$$

For most dusty plasmas $\omega_{Jd}^2<\omega_{pd}^2$ and the deviation from charge neutrality is always negative, even if small. Remark that the electrons and ions need not be Boltzmann distributed, that was not even used here! As a matter of fact, any hypothesis that algebraically relates φ_0 and ψ_0 allows a similar elimination of the Laplacians between the two Poisson's equations, and will lead to analogous conclusions.

Charge non-neutrality will be most prominent in the central region, which is evident from both a physical and a mathematical point of view. Physically speaking, the (negatively charged) heavy dust grains tend to concentrate in the central region, and this mass gathering can only be balanced by (weak) dust pressure and a stronger outward electrostatic pull, imparted by enough positive charges farther away.

In mathematical terms, the electrostatic potential φ_0 and its first derivative $d\varphi_0/dz$ vanish for physical reasons at the central location. Imposing there strict charge neutrality means that from the electrostatic Poisson's equation also the second derivative $d^2\varphi_0/dz^2$ vanishes, and then the function φ_0 becomes identically zero everywhere. Without gravitation or imposed inhomogeneities, that is why normal plasmas can have uniform equilibria that are perfectly charge neutral throughout. However, in the presence of self-gravitation $\varphi_0=0$ would impose that also ψ_0 be zero, or at least equal to an adjustable constant. This clearly is not allowed by the gravitational Poisson's equation, unless the Jeans swindle creeps in through the back door.

V. ASTROPHYSICAL APPLICATIONS

In the models where we could obtain explicit analytical solutions, the inhomogeneity length scales are of the order of the corresponding Jeans lengths. In order to get a feeling for the sizes involved, we will give some simple numerical estimates for the different cases, based on numbers for interstellar dust clouds [1,2]. These are dust formation regions where a hydrogen plasma is present and hence all or part of the dust can be taken as charged. We assume that the interstellar dust has a typical density of $n_{d00}=10^{-1}$ m^{-3}, an average mass for water-ice, micron-sized grains of $m_d=4\times10^{-15}$ kg and carries $Z_d=1500$ electron charges per grain. This equilibrium charge follows from the simple primary charging model [7,8], for plasma temperatures of $T_i\simeq T_e=10^4$ K. At $n_{i00}=5\times10^3$ m^{-3} and $n_{e00}=4.85\times10^3$ m^{-3}, the global Debye length is $\lambda_D=70$ m, obtained from $\lambda_D^{-2}=\lambda_{De}^{-2}+\lambda_{Di}^{-2}$. There is thus the standard ordering between the dust grain size $a=10^{-6}$ m, the average intergrain distance $d\simeq2$ m related to the dust density by $n_{d00}d^3\sim1$, and $\lambda_D=70$ m, so that $a\ll d\leqslant\lambda_D$ and we can indeed speak of a dusty plasma rather than charged dust in plasma [7,8].

The interstellar magnetic field is of the order of $B_0=3\times10^{-10}$ T, and the dust temperature is $T_d=30$ K. Such parameters result in a dust Jeans frequency of $\omega_{Jd}=5.8\times10^{-13}$ s^{-1} and a dust thermal velocity of $v_{td}=3.2\times10^{-4}$ m/s. If the dust were neutral, this would give a length of $\lambda_{nd}=v_{td}/\omega_{Jd}=5.5\times10^5$ km, less than half a percent of an astronomical unit.

On the other hand, if the dust is charged as indicated, the dust-acoustic velocity is $c_{da}=2.8\times10^{-2}$ m/s, clearly showing the preponderance of the plasma pressure in rendering the collapse of the heavy dust more difficult. The corresponding length now is $\lambda_{da}=c_{da}/\omega_{Jd}=4.8\times10^7$ km, a third of an astronomical unit.

Finally, for strictly transverse instabilities, the global Alfvén velocity is $V_A=13.4$ m/s, showing the even larger influence of magnetic effects in countering collapse. The corresponding length scales hence are $\lambda_{ms}\simeq V_A/\omega_{Jd}=2.3\times10^{10}$ km, some 150 astronomical units, about the size of the solar heliosphere in the upstream direction, towards the heliopause.

The conclusions of this simple exercise clearly point to the interaction of the charged dust with the hotter plasma and (or) a transverse magnetic field as important antagonists of gravitational collapse. Of course, the rather large unreliability of the present interstellar data about (charged) dust cautions us to take the above numbers with a generous pinch of salt, but nevertheless the message is pointing in the right direction, as far as our theoretical developments are concerned.

VI. SUMMARY

To conclude, we have discussed several aspects of self-gravitation in dusty space plasmas, when the interplay between self-gravitation and electromagnetic effects becomes important for heavier charged dust grains. We discussed some of the possible stationary states and determined the scales on which the system varies for those cases where explicit solutions can be obtained. Surprisingly enough, the inhomogeneity lengths are of the order of the Jeans lengths determined in the usual way by studying local perturbations, as if the stationary state were uniform, in other words, by applying what is called the Jeans swindle. In this sense, we will turn the argument around and use the Jeans swindle or local perturbation approach to find the scales over which the system itself is no longer homogeneous, when no explicit stationary configurations can be obtained. To get a feeling for possible orders of magnitude, these findings were applied to interstellar dust clouds, where, however, the paucity of reliable data cautions against taking the scale lengths too literally.

ACKNOWLEDGMENTS

F. Verheest thanks the Bijzonder Onderzoeksfonds (Universiteit Gent) and the Fonds voor Wetenschappelijk Onderzoek (Vlaanderen) for research grants. V.M. Čadež acknowledges support by the Belgian Institute for Space Aeronomy and by the Belgian Federal Office for Scientific, Technical, and Cultural Affairs.

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